

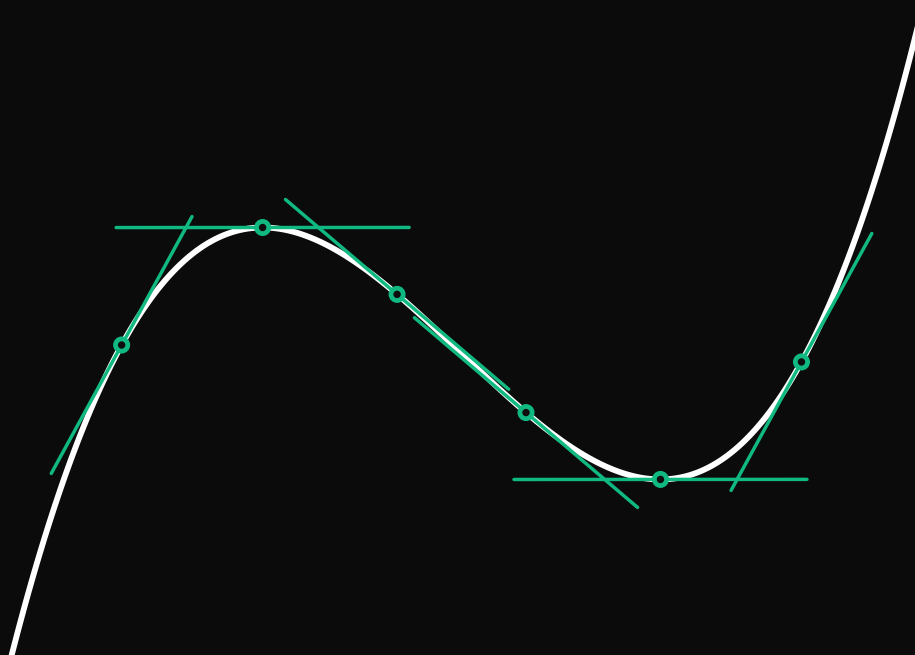
Hence

IB Mathematics · Analysis and approaches · SL

Free sample

Differential calculus.

Six exam-style questions. Every mark explained.



6

questions, Paper 1 and
Paper 2 style

52

marks, full IB-style
markscheme

3

levels, from routine to
grade 7

2

calculators: TI-Nspire CX
II, NumWorks

Written and verified by a practising IB mathematics teacher, produced with AI assistance. Not affiliated with or endorsed by the International Baccalaureate Organization.

About this booklet

Contents	Questions 2 and 5 (Paper 1 style, Section A) Question 14 (Paper 1 style, Section B) Questions 18 and 22 (Paper 2 style, Section A) Question 25 (Paper 2 style, Section B)
Syllabus	SL 5.1, 5.2, 5.3, 5.4, 5.6, 5.7, 5.8
Calculators	Paper 1 style: no calculator. Paper 2 style: graphic display calculator.
Numbering	Question numbers are those of the full pack (28 questions, 4 papers' worth of practice).

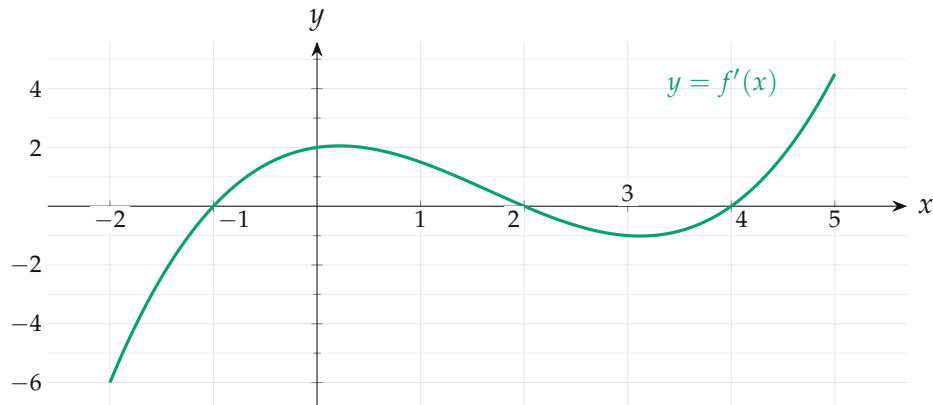
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5**5 marks** SL 5.2 · 5.8

●●○ Typical

The following diagram shows the graph of $y = f'(x)$, the derivative of a function f , for $-2 \leq x \leq 5$. The graph of f' crosses the x -axis at $x = -1$, $x = 2$ and $x = 4$.



- (a) Write down the x -coordinates of the points on the graph of f at which the tangent is horizontal. [1]
- (b) Write down the intervals on which f is increasing. [2]
- (c) Find the x -coordinate of the local maximum point of the graph of f . Justify your answer. [2]

Paper 1 style · Section B

No calculator

14 14 marks SL 5.6 · 5.7 · 5.8

●●○ Typical to demanding

Let $f(x) = (x^2 - 3)e^x$, for $x \in \mathbb{R}$.

- (a) Show that $f'(x) = (x + 3)(x - 1)e^x$. [3]
- (b) Find the x -coordinates of the stationary points of the graph of f . [2]
- (c) Show that the graph of f has a local maximum at $x = -3$, and find the exact value of this local maximum. [3]

It is given that $f''(x) = (x^2 + 4x - 1)e^x$.

- (d) Find the exact x -coordinates of the points of inflexion of the graph of f . Justify your answer. [3]

It is also given that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$.

- (e) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. [3]

Answer this question on separate paper.

Paper 2 style · Section A

Graphic display calculator

18**6 marks**

SL 5.1 · 5.6

●○○ Routine

A cup of tea is left to cool in a room. The temperature of the tea, in °C, t minutes after it is poured, is modelled by

$$T(t) = 21 + 64e^{-0.085t}, \quad t \geq 0.$$

- (a) Write down the temperature of the tea when it is poured. [1]
- (b) Find $T'(10)$. [2]
- (c) Interpret your answer to part (b) in the context of the question. [1]
- (d) Find the time at which the temperature of the tea is decreasing at a rate of 1 °C per minute. [2]

22 6 marks SL 5.4

... Demanding

Let $f(x) = e^{x/2} - x$. The point $P(0, 1)$ lies on the graph of f .

(a) Find the equation of the normal to the graph of f at P . [3]

The normal meets the graph of f again at the point Q .

(b) Find the x -coordinate of Q . [3]

Paper 2 style · Section B

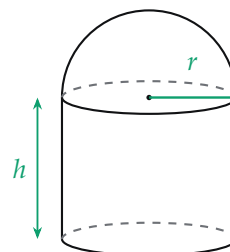
Graphic display calculator

25**15 marks** SL 5.8

●●● Typical to demanding

A sensor housing is made from thin sheet metal. It consists of a cylinder of radius r cm and height h cm, closed at the base, with a hemispherical cap of radius r cm on top, as shown in the diagram. There is no metal between the cylinder and the cap.

The volume enclosed by the housing is 500 cm^3 .



(a) Show that $h = \frac{500}{\pi r^2} - \frac{2r}{3}$. [2]

(b) Show that the total external surface area, $S \text{ cm}^2$, of the housing is given by $S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$. [3]

(c) Find $\frac{dS}{dr}$. [2]

(d) Find the value of r for which S is a minimum. Justify that it is a minimum. [3]

(e) Show that, for this value of r , the height of the cylinder is equal to its radius. [2]

The housing must fit inside a tube, so that its radius can be at most 4 cm.

(f) Find the least possible surface area of a housing that fits inside the tube. Justify your answer. [3]

Answer this question on separate paper.

Answers

Final answers only — the teacher edition gives the full markscheme

2 (a) -2 (b) $y = \frac{1}{2}x + \frac{9}{2}$ (c) $(-9, 0)$

5 (a) $-1, 2, 4$ (b) $-1 < x < 2$ and $4 < x \leq 5$ (c) $x = 2$ (f' changes from positive to negative)

14 (b) $-3, 1$ (c) $6e^{-3}$ (d) $-2 \pm \sqrt{5}$ (e) $0 < k < 6e^{-3}$

18 (a) 85°C (b) -2.33 (c) decreasing at 2.33°C per minute after 10 minutes (d) 19.9 minutes

22 (a) $y = 2x + 1$ (b) 5.84

25 (c) $\frac{10}{3}\pi r - \frac{1000}{r^2}$ (d) $r = 4.57$ (f) 334 cm^2 (when $r = 4$)