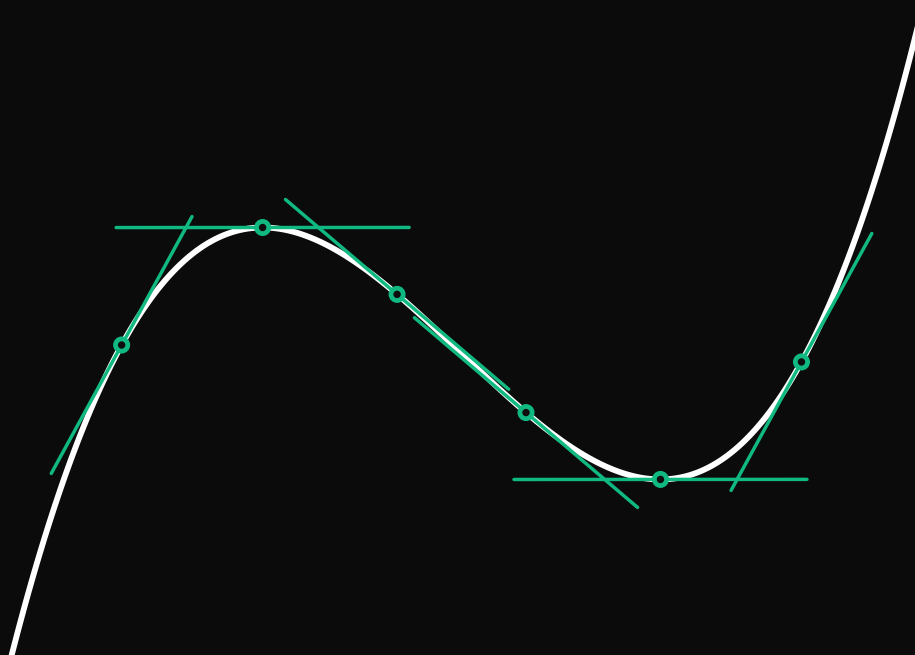


Free sample · Teacher edition

Differential calculus.

Six exam-style questions. Every mark explained.



6

questions, Paper 1 and
Paper 2 style

52

marks, full IB-style
markscheme

3

levels, from routine to
grade 7

2

calculators: TI-Nspire CX
II, NumWorks

Written and verified by a practising IB mathematics teacher, produced with AI assistance. Not affiliated with or endorsed by the International Baccalaureate Organization.

About this booklet

Contents	Questions 2 and 5 (Paper 1 style, Section A) Question 14 (Paper 1 style, Section B) Questions 18 and 22 (Paper 2 style, Section A) Question 25 (Paper 2 style, Section B)
Syllabus	SL 5.1, 5.2, 5.3, 5.4, 5.6, 5.7, 5.8
Calculators	Paper 1 style: no calculator. Paper 2 style: graphic display calculator. Key sequences are given for the TI-Nspire CX II and the NumWorks.
Marks	M = method; A = accuracy, normally dependent on the preceding M; R = reasoning; AG = answer given in the question (no mark); FT = follow through from an earlier error. Marks in brackets, e.g. (A1), may be implied by later correct work.
Numbering	Question numbers are those of the full pack (28 questions, 4 papers' worth of practice).

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Paper 1 style · Section A

No calculator

2 6 marks SL 5.4

●●○ Typical

Let $f(x) = 3 + 4x - x^2$. The point $P(3, 6)$ lies on the graph of f .

(a) Find the gradient of the tangent to the graph of f at P . [2]

(b) Find the equation of the normal to the graph of f at P . Give your answer in the form $y = mx + c$. [3]

The normal meets the x -axis at the point Q .

(c) Find the coordinates of Q . [1]

MARKSCHEME (a)

$$f'(x) = 4 - 2x$$

(A1)

$$\text{gradient at } P = f'(3) = -2$$

A1

Total 2 marks

MARKSCHEME (b)

$$\text{gradient of the normal} = \frac{1}{2}$$

(A1)

$$y - 6 = \frac{1}{2}(x - 3) \quad \text{OR} \quad 6 = \frac{1}{2}(3) + c$$

M1

$$y = \frac{1}{2}x + \frac{9}{2}$$

A1

Total 3 marks

Note Award M1 for substituting the coordinates of P with *their* normal gradient. Award M0 if the tangent gradient -2 is used. FT from their answer to (a).

MARKSCHEME (c)

$$Q(-9, 0)$$

A1

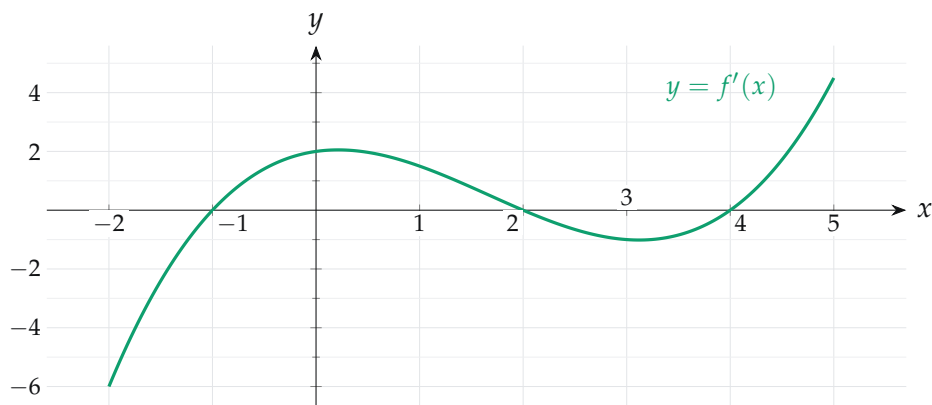
Total 1 mark

Note FT from their equation in (b). Accept $x = -9$, $y = 0$.

5**5 marks** SL 5.2 · 5.8

●●○ Typical

The following diagram shows the graph of $y = f'(x)$, the derivative of a function f , for $-2 \leq x \leq 5$. The graph of f' crosses the x -axis at $x = -1$, $x = 2$ and $x = 4$.



- (a) Write down the x -coordinates of the points on the graph of f at which the tangent is horizontal. [1]
- (b) Write down the intervals on which f is increasing. [2]
- (c) Find the x -coordinate of the local maximum point of the graph of f . Justify your answer. [2]

MARKSCHEME (a) $x = -1, x = 2, x = 4$ **A1**

Total 1 mark

Note All three values are needed for A1.**MARKSCHEME (b)** $-1 < x < 2$ **A1** $4 < x \leq 5$ **A1**

Total 2 marks

Note Accept $4 < x < 5$ and $x > 4$, and accept non-strict inequalities at -1 , 2 and 4 . Accept interval notation. Award A1A0 if an extra, incorrect interval is given.

MARKSCHEME (c) f' changes sign from positive to negative at $x = 2$ **R1** $x = 2$ **A1**

Total 2 marks

Note Do not award R0A1. A reason based only on $f'(2) = 0$ earns R0: f' is also zero at the two local minimum points $x = -1$ and $x = 4$.

Paper 1 style · Section B

No calculator

14

14 marks SL 5.6 · 5.7 · 5.8

●●● Typical to demanding

Let $f(x) = (x^2 - 3)e^x$, for $x \in \mathbb{R}$.

- (a) Show that $f'(x) = (x + 3)(x - 1)e^x$. [3]
- (b) Find the x -coordinates of the stationary points of the graph of f . [2]
- (c) Show that the graph of f has a local maximum at $x = -3$, and find the exact value of this local maximum. [3]

It is given that $f''(x) = (x^2 + 4x - 1)e^x$.

- (d) Find the exact x -coordinates of the points of inflexion of the graph of f . Justify your answer. [3]

It is also given that $f(x) \rightarrow 0$ as $x \rightarrow -\infty$.

- (e) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. [3]

MARKSCHEME (a)

attempt to use the product rule

M1

$$f'(x) = 2xe^x + (x^2 - 3)e^x$$

A1

$$= (x^2 + 2x - 3)e^x$$

A1

$$= (x + 3)(x - 1)e^x$$

AG

Total 3 marks

Note The intermediate form $(x^2 + 2x - 3)e^x$, or equivalent, must be seen for the second A1.**MARKSCHEME (b)**

$$f'(x) = 0 \text{ and } e^x > 0 \text{ (or } e^x \neq 0\text{), so } (x + 3)(x - 1) = 0$$

(M1)

$$x = -3, x = 1$$

A1

Total 2 marks

MARKSCHEME (c)

EITHER

sign of f' : positive for $x < -3$, negative for $-3 < x < 1$ (sign diagram or words)

M1

 f' changes from positive to negative at $x = -3$, so local maximum

R1

OR

$$f''(-3) = (9 - 12 - 1)e^{-3} = -4e^{-3}$$

M1

 $f''(-3) < 0$, so local maximum

R1

THEN

local maximum value $f(-3) = 6e^{-3}$

A1

Total 3 marks

Note Accept $\frac{6}{e^3}$. A decimal value only earns A0.

MARKSCHEME (d)

$$f''(x) = 0 \Rightarrow x^2 + 4x - 1 = 0 \text{ (since } e^x > 0 \text{)}$$

M1

$$x = -2 \pm \sqrt{5}$$

A1

$x^2 + 4x - 1$ has two distinct roots, so f'' changes sign (concavity changes) at each of them

R1

Total 3 marks

Note Accept a correct sign diagram of f'' for R1. Do not award R1 for " $f''(x) = 0$ " alone.

MARKSCHEME (e)

f increases from 0 to $6e^{-3}$ for $x < -3$, decreases to $-2e$ for $-3 < x < 1$, then increases without bound (words or a sketch)

R1

$$k > 0$$

A1

$$k < 6e^{-3}$$

A1

Total 3 marks

Note Both inequalities must be strict. Accept $0 < k < \frac{6}{e^3}$ and interval notation such as $]0, 6e^{-3}[$. A candidate who includes $k = 0$ or negative values of k has not used the fact that f increases from 0 for $x < -3$: award R0A0A1 at most. FT from their local maximum in (c).

Paper 2 style · Section A

Graphic display calculator

18

6 marks SL 5.1 · 5.6

●○○ Routine

A cup of tea is left to cool in a room. The temperature of the tea, in °C, t minutes after it is poured, is modelled by

$$T(t) = 21 + 64e^{-0.085t}, \quad t \geq 0.$$

- (a) Write down the temperature of the tea when it is poured. [1]
- (b) Find $T'(10)$. [2]
- (c) Interpret your answer to part (b) in the context of the question. [1]
- (d) Find the time at which the temperature of the tea is decreasing at a rate of 1 °C per minute. [2]

MARKSCHEME (a)

85 °C

A1

Total 1 mark

MARKSCHEME (b)

attempt to differentiate, or use of GDC

(M1)

 $T'(10) = -2.33$ (−2.32513...)

A1

Total 2 marks

MARKSCHEME (c)

10 minutes after it is poured, the temperature of the tea is decreasing at about 2.33 °C per minute

R1

Total 1 mark

Note Both the idea of decreasing and a rate in °C per minute are needed. FT from their answer to (b).

MARKSCHEME (d)

 $T'(t) = -1$ OR $5.44e^{-0.085t} = 1$

(M1)

 $t = 19.9$ minutes (19.9268...)

A1

Total 2 marks

Note Award M1 for $T'(t) = 1$ only if followed by a correct interpretation of the sign.

ON THE CALCULATOR

TI-Nspire CX II	(b) Calculator: menu → 4 Calculus → 1 Numerical Derivative at a Point, Value: 10, then type $21 + 64e^{-0.085x}$ in the template → -2.32514 . (d) Differentiate by hand: $T'(t) = -5.44e^{-0.085t}$. Graph: $f_1(x) = -5.44e^{-0.085x}$, $f_2(x) = -1$; menu → 6 Analyze Graph → 4 Intersection → $x = 19.9268$.
NumWorks	(b) Calculation: Toolbox → Calculus → Derivative, type $21 + 64e^{-0.085x}$, then the value 10 → -2.325137 . (d) With $T'(t) = -5.44e^{-0.085t}$: Functions: $f(x) = -5.44e^{-0.085x}$, $g(x) = -1$; Graph → Calculate → Find → Intersection → $x = 19.92681$.

22**6 marks** SL 5.4

●●● Demanding

Let $f(x) = e^{x/2} - x$. The point $P(0, 1)$ lies on the graph of f .

(a) Find the equation of the normal to the graph of f at P . [3]

The normal meets the graph of f again at the point Q .

(b) Find the x -coordinate of Q . [3]

MARKSCHEME (a)

$$f'(x) = \frac{1}{2}e^{x/2} - 1, \text{ so } f'(0) = -\frac{1}{2}$$

$$\text{normal: } y = 2x + 1$$

(M1)A1

A1

Total 3 marks

Note Award M1 for an attempt to differentiate, or for using the GDC to find $f'(0)$.

MARKSCHEME (b)

$$e^{x/2} - x = 2x + 1 \text{ (or a sketch of both graphs)}$$

(M1)

recognising that $x = 0$ is P , so the other solution is needed

(R1)

$$x = 5.84 \text{ (5.83660...)}$$

A1

Total 3 marks

Note FT from their normal in (a), provided it meets the graph of f again. Award (M1)(R0)A0 for $x = 0$ only.

ON THE CALCULATOR

TI-Nspire CX II	Graph: $f_1(x) = e^{x/2} - x$, $f_2(x) = 2x + 1$; menu → 4 Window/Zoom → 1 Window Settings: x from -2 to 8 , y from -2 to 20 . menu → 6 Analyze Graph → 4 Intersection, lower bound 1, upper bound 8 → $x = 5.8366$.
NumWorks	Functions: $f(x) = e^{x/2} - x$, $g(x) = 2x + 1$. Graph → Axes → Values of X: minimum 1, maximum 8 (so that the search starts after P). Graph → Calculate → Find → Intersection → $x = 5.836601$.

Paper 2 style · Section B

Graphic display calculator

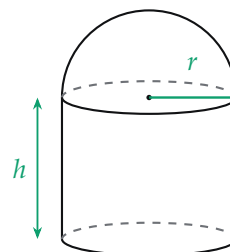
25

15 marks SL 5.8

●●● Typical to demanding

A sensor housing is made from thin sheet metal. It consists of a cylinder of radius r cm and height h cm, closed at the base, with a hemispherical cap of radius r cm on top, as shown in the diagram. There is no metal between the cylinder and the cap.

The volume enclosed by the housing is 500 cm^3 .



(a) Show that $h = \frac{500}{\pi r^2} - \frac{2r}{3}$. [2]

(b) Show that the total external surface area, $S \text{ cm}^2$, of the housing is given by $S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$. [3]

(c) Find $\frac{dS}{dr}$. [2]

(d) Find the value of r for which S is a minimum. Justify that it is a minimum. [3]

(e) Show that, for this value of r , the height of the cylinder is equal to its radius. [2]

The housing must fit inside a tube, so that its radius can be at most 4 cm.

(f) Find the least possible surface area of a housing that fits inside the tube. Justify your answer. [3]

MARKSCHEME (a)

$$\pi r^2 h + \frac{2}{3}\pi r^3 = 500$$

$$h = \frac{500}{\pi r^2} - \frac{2r}{3}$$

M1A1

AG

Total 2 marks

Note M1 for adding the volumes of a cylinder and of a hemisphere; A1 for the correct equation. The AG line must follow from correct working.

MARKSCHEME (b)

$$S = \pi r^2 + 2\pi r h + 2\pi r^2 \text{ (base, curved surface, hemisphere)}$$

$$2\pi r \left(\frac{500}{\pi r^2} - \frac{2r}{3} \right) = \frac{1000}{r} - \frac{4}{3}\pi r^2$$

$$S = \frac{5}{3}\pi r^2 + \frac{1000}{r}$$

M1A1

A1

AG

Total 3 marks

Note M1 for a sum of three areas; A1 for all three correct, including $2\pi r^2$ for the hemisphere.

MARKSCHEME (c)

$$\frac{dS}{dr} = \frac{10}{3}\pi r - \frac{1000}{r^2}$$

A1A1

Total 2 marks

MARKSCHEME (d)

$$\frac{10}{3}\pi r - \frac{1000}{r^2} = 0, \text{ or the minimum of } S \text{ on the GDC}$$

(M1)

$$r = 4.57 \text{ (4.57078...)}$$

A1

EITHER

$$\frac{d^2S}{dr^2} = \frac{10}{3}\pi + \frac{2000}{r^3} > 0 \text{ for } r > 0, \text{ so minimum}$$

R1

OR

$$\frac{dS}{dr} \text{ changes from negative to positive at } r = 4.57, \text{ so minimum}$$

R1

Total 3 marks

Note Accept a GDC graph of S with a clearly identified minimum point for R1.

MARKSCHEME (e)

$$\text{from (d), } \frac{10}{3}\pi r^3 = 1000, \text{ so } \pi r^3 = 300$$

M1

$$h = \frac{500}{\pi r^2} - \frac{2r}{3} = \frac{500r}{\pi r^3} - \frac{2r}{3} = \frac{5r}{3} - \frac{2r}{3}$$

A1

$$h = r$$

AG

Total 2 marks

Note A numerical check only (e.g. substituting $r = 4.57$ and obtaining $h \approx 4.57$) earns M0A0: the question asks for an exact argument.

MARKSCHEME (f)

$$\frac{dS}{dr} < 0 \text{ for } 0 < r < 4.57, \text{ so } S \text{ is decreasing for } 0 < r \leq 4$$

R1

$$\text{so the least value is at } r = 4: S = \frac{5}{3}\pi(16) + \frac{1000}{4}$$

M1

$$S = 334 \text{ cm}^2 \text{ (333.776...)}$$

A1

Total 3 marks

Note Award R1 for any valid argument that the least value on $0 < r \leq 4$ occurs at $r = 4$, including a GDC graph of S restricted to this interval. An answer of 328 (the unconstrained minimum) earns R0M0A0.

ON THE CALCULATOR

TI-Nspire CX II

(d) Graph: $f1(x) = \frac{5}{3}\pi x^2 + \frac{1000}{x}$; menu → 4 Window/Zoom → 1 Window Settings: x from 0 to 10, y from 0 to 600. menu → 6 Analyze Graph → 2 Minimum, bounds 1 and 9 → (4.57078, 328.171).

NumWorks

(d) Functions: $f(x) = \frac{5}{3}\pi x^2 + \frac{1000}{x}$. Graph → Axes → Values of X : 0 to 10; Values of Y : 0 to 600. Graph → Calculate → Find → Minimum → $x = 4.570781$, $f(x) = 328.1710$.