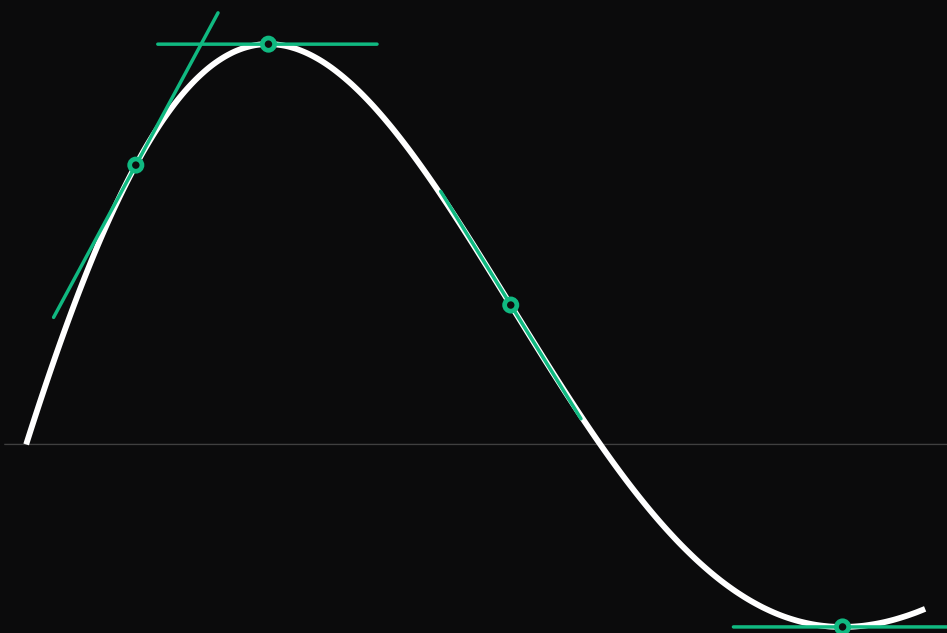


Free guide

Differential calculus on your GDC.

Every calculator move Paper 2 asks for, on the TI-Nspire CX II and the NumWorks.

**7**

moves, in the order of the course

2

calculators, side by side at every step

3

exam-style questions, worked

16

calculator screens, drawn step by step

How to use this guide

What it covers	The seven calculator moves that differential calculus questions require in Paper 2: SL 5.1, 5.4, 5.7, 5.8 and 5.9 of the IB Mathematics: analysis and approaches guide. In Paper 1, no calculator is allowed.
How it works	Each move starts from an exam-style question. You see what to write <i>before</i> you touch the calculator, the keys on both calculators side by side, the screen you should reach, and what to write <i>after</i> .
Keys	 a key on the calculator Find a menu item or on-screen button  something you type ► then
Calculators	NumWorks with software version 23 or later; TI-Nspire CX II or CX II-T (non-CAS) in the Scratchpad. Screens are simplified drawings: they show where you should be, not every pixel. Menu items are named, not numbered, because numbers change between software versions.
Answers	Values on the screens are those the calculator displays: 10 significant figures in the NumWorks Calculation app, 7 in its graphs, 6 on the TI-Nspire set to Float 6. Answers are then given to 3 significant figures, as the IB requires unless a question says otherwise.

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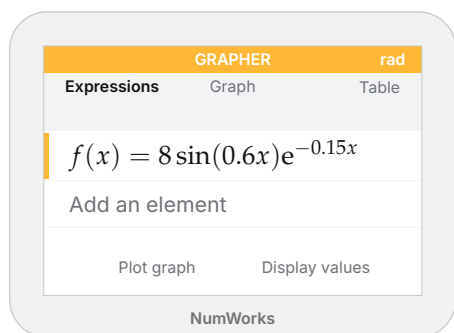
0 BEFORE YOU START

Radians, precision, and the function stored once

Trigonometric functions in calculus are always in radians: a calculator left in degrees gives wrong gradients without any warning. Store the function once, so that every later move can call it by its name instead of retyping it.

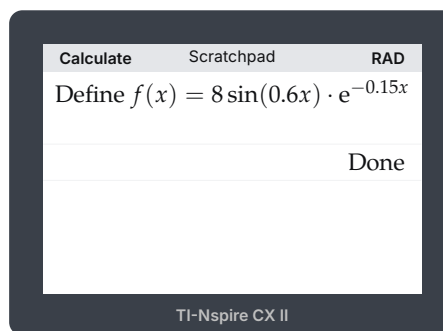
NUMWORKS

- 1 **home** ► **Settings** **OK** ► **Angle measure** ► **Radians** **OK**.
- 2 **home** ► **Grapher** **OK**. On **Add an element**, press **OK**, type $8\sin(0.6x)e^{-0.15x}$ and press **EXE**. The function is named f .



TI-NSPIRE CX II

- 1 **home** ► **Settings** ► **Document Settings:** **Angle Radian**, **Display Digits Float 6** ► **Make Default**.
- 2 **home** ► **Calculate**. **menu** ► **Actions** ► **Define**, type $f(x)=8\sin(0.6x)e^{-0.15x}$ and press **enter**.



IN THE EXAM Check the angle setting at the start of Paper 2: *rad* on the NumWorks, *RAD* in the TI-Nspire tab bar. Check again once exam mode is on: entering it can reset the settings.

1

MOVE 1 · SL 5.1 · 5.4

Find the gradient at a point: the numerical derivative

The question used for moves 1 to 5

Let $f(x) = 8 \sin(0.6x) e^{-0.15x}$, for $0 \leq x \leq 8$.

- Find the gradient of the graph of f at $x = 1$. [2]
- Find the equation of the tangent and the equation of the normal to the graph of f at $x = 1$. [4]
- Find the coordinates of the local maximum point and of the local minimum point of the graph of f . [3]
- Find the values of x for which the gradient of the graph of f is -2 . [3]
- The graph of f has a point of inflexion P . Find the coordinates of P . [3]

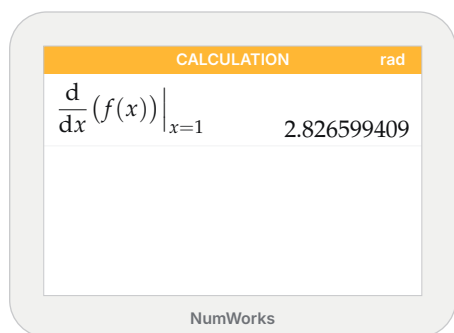
Question (a) Find the gradient of the graph of f at $x = 1$. [2]

ON YOUR PAPER · BEFORE THE CALCULATOR

The gradient at $x = 1$ is $f'(1)$. Write it down in this notation: it shows the examiner which value you are about to compute.

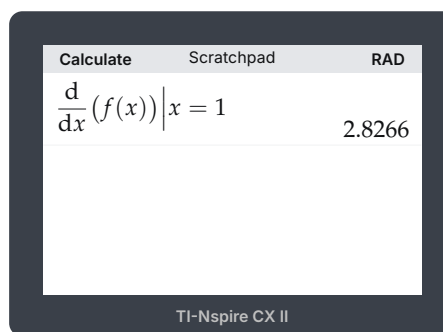
NUMWORKS

- home** ► **Calculation** **OK**.
- toolbox** ► **Calculus** ► **Derivative**.
- Type $f(x)$, press **►** until the cursor reaches the value box, type 1 , then **EXE**.



TI-NSPIRE CX II

- In Calculate: **menu** ► **Calculus** ► **Numerical Derivative at a Point**.
- Value:** 1 , then **enter**.
- In the template, type $f(x)$ and press **enter**.



ON YOUR PAPER · AFTER THE CALCULATOR

$$f'(1) = 2.83 \quad (2.82659\dots)$$

IN THE EXAM Write $f'(1) = 2.83$, never the calculator syntax ($nDeriv, diff$): it is not accepted as working. Keep the unrounded value in the calculator for the next part.

2 MOVE 2 · SL 5.4

Find the equations of the tangent and of the normal at a point

Question (b) Find the equation of the tangent and the equation of the normal to the graph of f at $x = 1$. [4]

ON YOUR PAPER · BEFORE THE CALCULATOR

A line needs a point and a gradient. Point: $(1, f(1))$. Tangent gradient: $m = f'(1)$, found in move 1.

Normal gradient: $-\frac{1}{m}$, because the normal is perpendicular to the tangent.

NUMWORKS

- 1 Straight after move 1, type $-1/$ then $\boxed{\text{ans}}$ and press $\boxed{\text{EXE}}$: the normal gradient.
- 2 Type $f(1)$ and press $\boxed{\text{EXE}}$: the y -coordinate of the point.

CALCULATION		rad
$\frac{d}{dx}(f(x)) _{x=1}$	2.826599409	
$-1/\text{ans}$	-0.3537820028	
$f(1)$	3.887938245	

NumWorks

TI-NSPIRE CX II

- 1 Straight after move 1, type $-1/$ then $\boxed{\text{ctrl}}$ $\boxed{(-)}$ for ans , and press $\boxed{\text{enter}}$.
- 2 Type $f(1)$ and press $\boxed{\text{enter}}$.

Calculate	Scratchpad	RAD
$\frac{d}{dx}(f(x)) _{x=1}$		2.8266
$\frac{-1}{\text{ans}}$		-0.353782
$f(1)$		3.88794

TI-Nspire CX II

ON YOUR PAPER · AFTER THE CALCULATOR

Tangent: $y - 3.88794 = 2.82660(x - 1)$, so $y = 2.83x + 1.06$.

Normal: $y - 3.88794 = -0.353782(x - 1)$, so $y = -0.354x + 4.24$.

IN THE EXAM The point-gradient form $y - y_1 = m(x - x_1)$ earns full marks unless the question asks for the form $y = mx + c$. Work with the unrounded values and round only the final coefficients: rounding $f(1)$ and $f'(1)$ first can shift the last digit.

3 MOVE 3 · SL 5.8

Find a local maximum or minimum on the graph

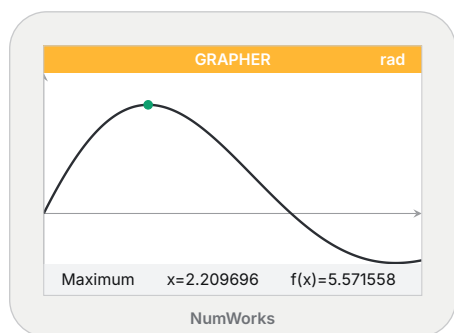
Question (c) Find the coordinates of the local maximum point and of the local minimum point of the graph of f . [3]

ON YOUR PAPER · BEFORE THE CALCULATOR

Nothing is required before the calculator here: in Paper 2, coordinates read from the graph earn the marks. Set the window to the domain, $0 \leq x \leq 8$, so that the calculator only searches where the question asks.

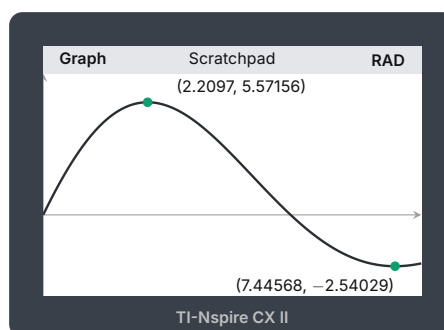
NUMWORKS

- 1 In Grapher: **Graph** tab. **Axes** ► **Values of X:** minimum **0**, maximum **8** ► **Confirm**.
- 2 **OK** on the curve ► **Find** ► **Maximum**.
- 3 Again: **OK** ► **Find** ► **Minimum**.



TI-NSPIRE CX II

- 1 **home** ► **Graph**. Type $f1(x)=f(x)$ and press **enter**.
- 2 **menu** ► **Window/Zoom** ► **Window Settings:** XMin **0**, XMax **8**, YMin **-4**, YMax **7**.
- 3 **menu** ► **Analyze Graph** ► **Maximum**. At *lower bound?* click left of the peak, at *upper bound?* right of it.
- 4 The same with **Minimum**.



ON YOUR PAPER · AFTER THE CALCULATOR

Local maximum $(2.21, 5.57)$; local minimum $(7.45, -2.54)$.

IN THE EXAM Give both coordinates, to 3 significant figures. If the window did not start at 0, the calculator could stop at a different turning point: always match the window to the domain.

4 MOVE 4 · SL 5.7 · 5.8

Find where the gradient takes a given value: the graph of f'

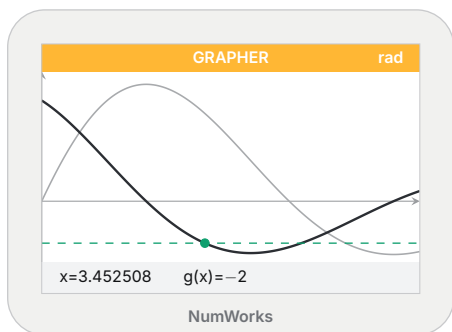
Question (d) Find the values of x for which the gradient of the graph of f is -2 . [3]

ON YOUR PAPER · BEFORE THE CALCULATOR

Write the equation $f'(x) = -2$: it shows your method, and markschemes usually reward it. The calculator then draws $y = f'(x)$, and you read where it takes the value -2 .

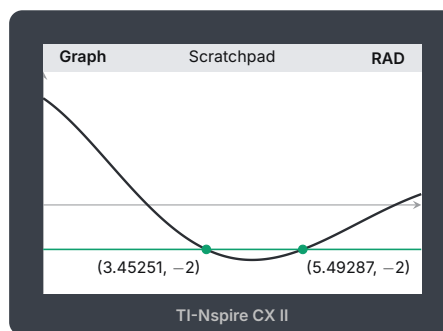
NUMWORKS

- 1 In Grapher, **Expressions** tab, **Add an element**: $g(x)=$, then **toolbox** ► **Calculus** ► **Derivative**, with $f(x)$ as the function and x as the value. **EXE**.
- 2 **Graph** tab. **Axes** ► **Values of Y**: -4 to 6 .
- 3 ▼ to select g , **OK** ► **Find** ► **X given Y**: -2 **OK**. Then ► for the next solution.



TI-NSPIRE CX II

- 1 In Graph, **tab** opens the entry line; ▲ back to f1 and replace it by $nDeriv(f(x),x)$ **enter**.
- 2 $f2(x)=-2$ **enter**. Window: YMin -4 , YMax 6 .
- 3 **menu** ► **Analyze Graph** ► **Intersection**, with bounds around each crossing.



ON YOUR PAPER · AFTER THE CALCULATOR

$f'(x) = -2$ when $x = 3.45$ or $x = 5.49$.

IN THE EXAM If the derivative can be found by hand, typing $f'(x)$ instead of the derivative template gives the same graph, and the written derivative may earn marks of its own. The graph of f' crosses $y = -2$ twice: a question asking for “the values” expects both.

5

MOVE 5 · SL 5.7 · 5.8

Find a point of inflexion: the extremum of f'

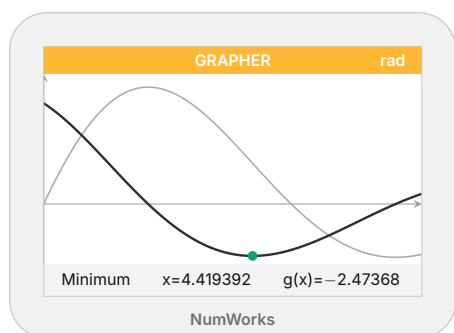
Question (e) The graph of f has a point of inflexion P . Find the coordinates of P . [3]

ON YOUR PAPER · BEFORE THE CALCULATOR

At a point of inflexion, $f''(x) = 0$ and f'' changes sign. Equivalently, f' has a local maximum or minimum there: the gradient stops decreasing and starts increasing. The graph of f' from move 4 is already on the screen, so look for its turning point.

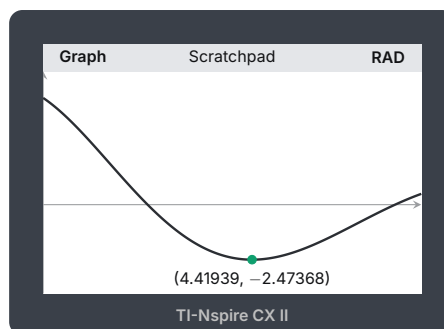
NUMWORKS

- 1 On g: **OK** ► **Find** ► **Minimum**.
- 2 **home** ► **Calculation**: type $f(4.419392)$ and press **EXE**.



TI-NSPIRE CX II

- 1 On f1: **menu** ► **Analyze Graph** ► **Minimum**, with bounds around the lowest point.
- 2 In Calculate, type $f(4.41939)$ and press **enter**.



ON YOUR PAPER · AFTER THE CALCULATOR

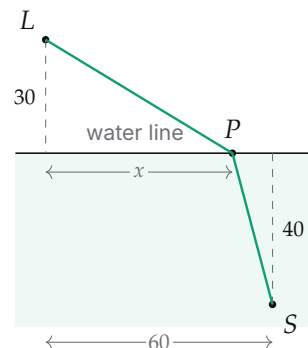
f' has a local minimum at $x = 4.42$, so f'' changes sign there (from negative to positive).
 $f(4.42) = 1.94$, so $P(4.42, 1.94)$.

IN THE EXAM The y -coordinate of P is a value of f , not of f' : -2.47 is the gradient at P , a common wrong answer. If the question asks you to justify that P is a point of inflexion, $f''(x) = 0$ alone is not enough: the reason " f'' changes sign" is what earns the mark.

6 MOVE 6 · SL 5.8

Optimisation: model, minimum, check the endpoints

A lifeguard at L stands on a straight beach, 30 m from the water line. A swimmer at S is 40 m out to sea and 60 m further along the shore. The lifeguard runs at 5 m s^{-1} to a point P on the water line, x m along, then swims at 1.5 m s^{-1} straight to S .



- (a) Show that the time taken, in seconds, is $T(x) = \frac{\sqrt{900 + x^2}}{5} + \frac{\sqrt{1600 + (60 - x)^2}}{1.5}$. [2]
- (b) Find the value of x that minimises the time, and the minimum time. [3]

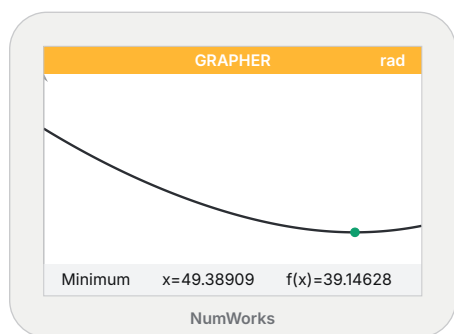
ON YOUR PAPER · BEFORE THE CALCULATOR

(a) is a “show that”: it is done by hand. $LP = \sqrt{30^2 + x^2}$ and $PS = \sqrt{40^2 + (60 - x)^2}$ by Pythagoras, and time = $\frac{\text{distance}}{\text{speed}}$ on each stretch.

(b) Write the domain, $0 \leq x \leq 60$, then find the minimum of T on the graph.

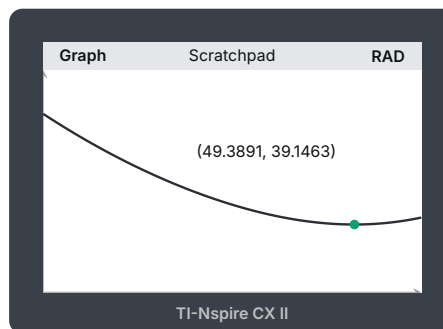
NUMWORKS

- In Grapher, delete f and g (select each line and press $\leftarrow$), then enter T as $f(x)$: the variable is always x .
- Axes:** Values of X 0 to 60, Values of Y 30 to 60.
- OK** ► **Find** ► **Minimum**.



TI-NSPIRE CX II

- In Graph, replace $f1$ by T and delete $f2$, so that only one graph is analysed.
- Window: XMin 0, XMax 60, YMin 30, YMax 60.
- menu** ► **Analyze Graph** ► **Minimum**, with bounds around the lowest point.



ON YOUR PAPER · AFTER THE CALCULATOR

$x = 49.4 \text{ m}$, and the minimum time is 39.1 s .

Check: $T(0) = 54.1 \text{ s}$ and $T(60) = 40.1 \text{ s}$ are both longer, so this local minimum is the least time on the domain.

IN THE EXAM Give the units and answer the question that was asked: here both x and the time. On a closed domain, compare the endpoints with the turning point; the least value is not always at a stationary point.

7

MOVE 7 · SL 5.9

Kinematics: at rest, acceleration, maximum speed

A particle moves in a straight line. Its velocity, in m s^{-1} , is $v(t) = 4 \cos(1.2t) - 0.5t$, for $0 \leq t \leq 6$, where t is in seconds.

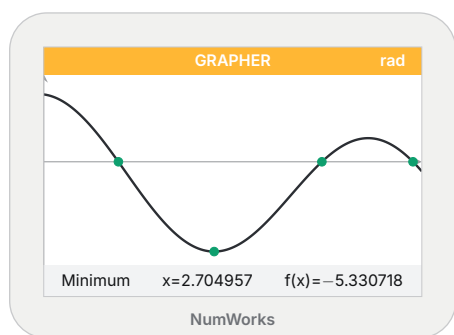
- Find the values of t for which the particle is at rest. [3]
- Find the acceleration of the particle when $t = 2$. [2]
- Find the maximum speed of the particle. [3]

ON YOUR PAPER · BEFORE THE CALCULATOR

(a) At rest: $v(t) = 0$. (b) Acceleration: $a(t) = v'(t)$, so find $v'(2)$. (c) Speed is $|v(t)|$: the maximum speed comes from the largest value *or* the most negative value of v , endpoints included.

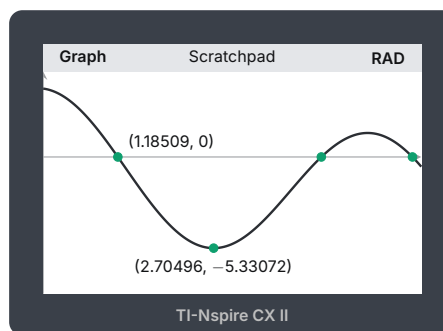
NUMWORKS

- In Grapher, replace f by v , with x for t : $4\cos(1.2x) - 0.5x$. **Axes:** x from 0 to 6, y from -8 to 5.
- OK** ► **Find** ► **Zeros**; ► for the next two.
- OK** ► **Find** ► **Minimum**.
- In Calculation, the derivative of v at 2 as in move 1.



TI-NSPIRE CX II

- In Graph, replace $f1$ by $4\cos(1.2x) - 0.5x$. Window: x from 0 to 6, y from -8 to 5.
- menu** ► **Analyze Graph** ► **Zero**, once for each crossing.
- menu** ► **Analyze Graph** ► **Minimum**.
- In Calculate, **Numerical Derivative at a Point** at 2, as in move 1.



ON YOUR PAPER · AFTER THE CALCULATOR

- $t = 1.19 \text{ s}, t = 4.41 \text{ s}, t = 5.86 \text{ s}$.
- $a(2) = v'(2) = -3.74 \text{ m s}^{-2}$ (calculator: -3.742223267 on the NumWorks, -3.74222 on the TI-Nspire).
- The largest value of v is $v(0) = 4$; the most negative is $v(2.70) = -5.33$. The maximum speed is 5.33 m s^{-1} .

IN THE EXAM The maximum *velocity* here is 4, at an endpoint; the maximum *speed* is 5.33. Reading the question's word carefully is worth the whole of part (c).

Now practise with 28 questions.

The full Hence pack on differential calculus: Paper 1 and Paper 2 style, a full IB-style markscheme for every mark, and these key sequences for every Paper 2 question.

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